

Normal versus inverted hierarchical models within μ - τ symmetry

N.Nimai Singh^{†,*1}, H. Zeen Devi[†] and Mahadev Patgiri[‡]

[†] *Department of Physics, Gauhati University, Guwahati-781014, India*

[‡] *Department of Physics, Cotton College, Guwahati-781001, India*

^{*} *The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 31014 Trieste, Italy.*

Abstract

We make a theoretical attempt to compare the predictions from normal and inverted hierarchical models, within the framework of $\mu - \tau$ symmetry. We consider three major theoretical issues in a self consistent ways, viz., predictions on neutrino mass and mixing parameters, stability under RG analysis in MSSM, and baryogenesis through leptogenesis. We further extend our earlier works on parametrisation of neutrino mass matrices obeying $\mu - \tau$ symmetry, using only two parameters in addition to an overall mass scale m_0 , to both normal and inverted hierarchy, and the ratio of these two parameters fixes the value of solar mixing angle. Such parametrisation though phenomenological, gives a firm handle on the analysis of the mass matrices and can also extend its prediction to lower values of solar mixing angle in the range $\tan^2 \theta_{12} = 0.50 - 0.35$. All predictions are in agreement with observed data except in one case, i.e., inverted hierarchy with opposite CP parity in the first two mass eigenvalues (Type B) with $m_3 \neq 0$, where Δm_{23}^2 is highly dependent on solar mixing angle, and the prediction is good for $\tan^2 \theta_{12} \leq 0.45$ only. We then check the stability of the inverted hierarchical model with opposite CP-parities, under radiative corrections in MSSM for large $\tan \beta \sim 58 - 60$ region and observe that the evolution of Δm_{21}^2 with energy scale, is highly dependent on the input-high scale value of solar mixing angle. Solar angles predicted by tri-bimaximal mixings angle and values

¹Regular Associate of ICTP.

E-mail address: nimai03@yahoo.com

lower than this, do not lead to the stability of the model at large $\tan \beta$ values. Similarly, the evolution of the atmospheric mixing angle with energy scale at large $\tan \beta$ values, shows sharp decrease with energy scale for the case with $m_3 \neq 0$. However, non-zero value of m_3 is essential to maintain the stability on the evolution of solar mass scale. We apply these mass matrices to estimate the baryon asymmetry of the Universe in a self consistent way and find that normal hierarchical model leads to the best result. Considering all these three pieces of theoretical investigations, we may conclude that normal hierarchical model is more favourable in nature.

PACS numbers: 14.60.Pq, 12.15.Ff, 13.15.+g, 13.40.Em

1 Introduction

The recent global 3ν oscillation analysis[1] indicates a mild departure from tribimaximal neutrino mixings. The decreasing trend in solar mixing is also consistent with the prediction from the Quark-Lepton Complementarity (QLC) relation [2,3,4] at the unification scale where the Cabibbo angle is taken at high scale[5]. In the theoretical front there are several attempts to find out the most viable models of neutrinos, and among them the μ - τ reflection symmetry[6-10] in neutrino mass matrix at high scale, has attracted considerable attentions in the last few years. Even the tri-bimaximal mixing[11,12] is a special case of this symmetry. It is expected that this symmetry has a strong potential to explain the present neutrino observed data[1].

The μ - τ symmetry leads to the maximal atmospheric mixing ($\theta_{23} = \pi/4$) and zero reactor angle ($\theta_{13} = 0$). The prediction on solar mixing angle θ_{12} remains arbitrary and it is generally fixed through a parametrisation in the mass matrix. This symmetry has the freedom to fix the solar mixing angle at lower values, even far below the tri-bimaximal value, without destroying the μ - τ symmetry. This is possible through the identification of a ratio of two parameters (referred to as flavor twister) present in the neutrino mass matrix and its subsequent variation in input values[13]. We are interested to parametrise both inverted as well as normal hierarchical neutrino mass models and then identify the flavor twistors responsible for lowering the solar mixing angle[14]. It is interesting to note that μ - τ symmetry gives a common origin for both hierarchical and inverted hierarchical neutrino mass matrices in agreement with latest data.

The μ - τ symmetry in neutrino mass matrix is assumed to hold in the charged lepton mass basis, although the charged lepton masses are obviously not μ - τ symmetric. However, such a scenario can be realised in gauge models with different Higgs doublets generating the up- and down-like particle masses[7,9,10,15,16]. A realisation of μ - τ symmetry in the flavor basis within the framework of SUSY SU(5) GUT, has strengthen the foundation of the symmetry as a full-fledged gauge theory[10]. We are now interested to investigate the phenomenological predictions of the μ - τ symmetry in neutrino sector and also possible application in leptogenesis[7,17]. In the present work we confine to analysis without phases, keeping our eyes on both predictions on neutrino masses and mixings consistent with latest data.

The paper is organised as follows. In section 2 we give a very brief overview on latest developments on μ - τ symmetry in neutrino mass mod-

els. In section 3 we give the parametrisation of the mass matrices for hierarchical and inverted hierarchical models in terms of only two parameters and identify the flavor twisters in both cases. This will be supplemented by detailed numerical analysis. In section 4 we discuss the stability under radiative corrections for large $\tan\beta$ values for inverted hierarchical model. We give a brief account on the predictions on baryogenesis using the same neutrino mass matrices. Section 6 concludes with a summary and discussion.

2 Neutrino mass matrices with μ - τ symmetry

The μ - τ symmetry in the neutrino mass matrix, implies an invariance under the simultaneous permutation of the second and third rows as well as the second and third columns in neutrino mass matrices[6],

$$m_{LL} = \begin{pmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{pmatrix}. \quad (1)$$

Neutrino mass matrix in eq.(1) predicts the maximal atmospheric mixing angle, $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. However the prediction on solar mixing angle θ_{12} is arbitrary, and it can be fixed by the input values of the parameters present in the mass matrix. Thus

$$\tan 2\theta_{12} = \left| \frac{2\sqrt{2}Y}{(X - Z - W)} \right| \quad (2)$$

which depends on four input parameters X, Y, Z and W . This makes us difficult to choose the values of these free parameters for a solution consistent with neutrino oscillation data. This point will be addressed in section 3 where the solar angle is made dependent only on the ratio of two parameters, η/ϵ . Such parametrization of the mass matrix enables us to analyse the neutrino mass matrix in a systematic and economical way[13]. The actual values of these two new parameters will be fixed by the data on neutrino mass squared differences.

The MNS leptonic mixing matrix U_{MNS} which diagonalises m_{LL} is defined by $m_{LL} = U_{MNS} D U_{MNS}^\dagger$ where $D = \text{diag.}(m_1, m_2, m_3)$, and

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \quad (3)$$

From the consideration of μ - τ reflection symmetry, U_{MNS} mixing matrix is generally parametrised by three rotations ($\theta_{23} = \pi/4$, $\theta_{13} = 0$):

$$U_{MNS} = O_{23}O_{13}O_{12} = O_{23}O_{12} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \\ s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad (4)$$

where $c_{12} = \cos \theta_{12}$, $s_{12} = \sin \theta_{12}$. Tri-bimaximal mixing (TBM) is a special case with $c_{12} = \sqrt{2/3}$ and $s_{12} = \sqrt{1/3}$,

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad (5)$$

where

$$O_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (6)$$

and

$$O_{12} = \begin{pmatrix} \sqrt{2/3} & -1/\sqrt{3} & 0 \\ 1/\sqrt{3} & \sqrt{2/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

For completeness we also give the three neutrino mass eigenvalues[9] corresponding to the neutrino mass matrix in eq.(1),

$$-m_1 = \frac{1}{2}[Z + W + X - \sqrt{8Y^2 + (Z + W - X)^2}], \quad (8)$$

$$m_2 = \frac{1}{2}[Z + W + X + \sqrt{8Y^2 + (Z + W - X)^2}], \quad (9)$$

$$m_3 = (Z - W). \quad (10)$$

The solar mixing angle is given by $\cos \theta_{12} = \sqrt{\frac{m_2+X}{m_1+m_2}}$, $\sin \theta_{12} = \sqrt{\frac{m_1-X}{m_1+m_2}}$. If $X = 0$, then we have a simple relation, $\tan^2 \theta_{12} = m_1/m_2$. The general form of mass matrix in eq.(1) can be fitted to both normal and inverted hierarchical models. Without changing the expression for the prediction on solar mixing angle in eq.(2), the parameters X , Z and W can be rearranged within the texture, giving many possible neutrino mass models obeying μ - τ symmetry. Some of these models are suitable for normal hierarchical model and other for inverted hierarchical model. This will be addressed in the next two subsections.

2.1 Normal hierarchical neutrino mass model

Many models based on normal hierarchy [8,9] make the 1-1 term (X) zero in the general neutrino mass matrix in eq.(1). This reduces one free parameter in the mass matrix. In the present analysis this can be done from the general form (1) by a mere rearrangement of parameters, which preserves the solar mixing angle given in eq.(2). Thus the mass matrix takes the new form

$$m_{LL} = \begin{pmatrix} 0 & Y & Y \\ Y & Z - X/2 & W - X/2 \\ Y & W - X/2 & Z - X/2 \end{pmatrix} \quad (11)$$

which can be simply expressed as[8,9],

$$m_{LL} = \begin{pmatrix} 0 & A & A \\ A & B & C \\ A & C & B \end{pmatrix}. \quad (12)$$

As discussed before, such mass matrix has interesting predictions on solar mixing angle in term of a ratio of first two neutrino mass eigenvalues[8,9,18,19],

$$\tan^2 \theta_{12} = \frac{m_1}{m_2}. \quad (13)$$

This form of neutrino mass matrix (12) is seesaw invariant[8] in the sense that both Dirac neutrino mass matrix m_{LR} and the right-handed Majorana mass matrix M_R also have the same form(12) of mass matrix. This model can be motivated[8] within the framework of SO(10) GUT where both quarks and charged leptons mass matrices have the broken μ - τ symmetry due to the presence of extra μ - τ antisymmetric parts in the mass matrices arising from **120** Higgs scalar, while mass mass matrices belonging to three neutrinos (m_{LL}, m_{LR}, M_R) are μ - τ symmetric due to **10** and **126** Higgs scalar in the SO(10)GUT model. In such model there is a correction in U_{MNS} from charged lepton sector.

Other interesting observations for the neutrino mass matrices (1) obeying μ - τ symmetry, can also be obtained from the seesaw formula using the general μ - τ symmetric Dirac neutrino mass matrix (1) in the diagonal basis of the right-handed Majorana mass matrix with two degenerate heavy mass eigenvalues (M_1, M_2, M_2) [7]. This is true only for normal hierarchical model and has a special application in resonant leptogenesis.

2.2 Inverted hierarchical neutrino mass model

In case of inverted hierarchical model, we assume that both CP even and CP odd in first two mass eigenvalues, have a common mass matrix[13]. The general form in eq.(1) with $Z \neq W$ will lead to small but $m_3 \neq 0$. For simplicity in the phenomenological analysis, one can push to $m_3 = 0$ condition without changing the expression on the prediction of solar mixing angle (2), through a simple rearrangement of parameters. The inverted hierarchical mass matrix has the form

$$m_{LL} = \begin{pmatrix} X & Y & Y \\ Y & (Z+W)/2 & (Z+W)/2 \\ Y & (Z+W)/2 & (Z+W)/2 \end{pmatrix}. \quad (14)$$

which can be rewritten as

$$m_{LL} = \begin{pmatrix} A & B & B \\ B & D & D \\ B & D & D \end{pmatrix}. \quad (15)$$

This obeys $\det(m_{LL}) = 0$ condition[20] and hence $m_3 = 0$. The above form (15) can now be expressed as [21],

$$m_{LL} = \begin{pmatrix} \delta_1 & 1 & 1 \\ 1 & \delta_2 & \delta_2 \\ 1 & \delta_2 & \delta_2 \end{pmatrix} m_0, \quad (16)$$

where $\delta_{1,2} < 1$ for inverted hierarchy with CP odd. Particular choices of the values of parameters (A, B, D) in (15) make the mass matrix either CP even ($B < D$) or CP odd ($B > D$) in first two mass eigenvalues ($m_1, \pm m_2, 0$), thus signifying a common origin for both mass models. Recently Babu et al [16] have presented a new realisation of inverted hierarchical mass matrix (16) based on $S_3 \times U(1)$ flavor symmetry where S_3 is the non-Abelian group generated by permutation of three objects, while the $U(1)$ is based for explaining the mass hierarchy of the leptons. In this construction the S_3 permutation symmetry is broken down to an Abelian S_2 in the neutrino sector, whereas it is broken completely in the charged lepton sector. The μ - τ symmetry is then realised in neutrino sector, while having non-degenerate charged leptons. The $U(1)$ symmetry acts as leptonic $L_e - L_\mu - L_\tau$ symmetry which is desirable for an inverted hierarchical model. The significant deviation of θ_{12} from $\pi/4$ comes from breaking of S_2 symmetry in charged lepton sector.

It can be pointed out here that the form of mass matrix (12) with 1-1 element zero in the mass matrix, can also be constructed for inverted hierarchy[22]. Since m_1 and m_2 are nearly degenerate in inverted hierarchy, this model leads to nearly bi-maximal mixings $\tan^2 \theta = m_1/m_2 \sim 0.98$, requiring large corrections from charged lepton sector to meet the data. Such models have problems and are not favoured by the recent data.

In a significant work by Mohapatra et al [10], the realization of μ - τ reflection symmetry in the neutrino mass matrix in the flavor basis (i.e. the basis where charged leptons are mass eigenstates), has been obtained in a realistic full-fledged gauge model based on $SUSY SU(5)GUT$ where leptons and quarks are treated together. In such model the requirement of μ - τ symmetry for neutrinos does not contradict with the observed fermion masses and mixings. The neutrino mass matrix having μ - τ symmetry, is assumed to arise from a triplet seesaw (type II) mechanism, which disentangles the neutrino flavor structure from quark flavor structure. The deviations of θ_{13} and θ_{23} from 0 and $\pi/4$ respectively, come from left-handed charged leptons mixing matrix.

3 Neutrino mass matrices in two parameters and numerical analysis

This section is the main part of the paper, where we are interested to express the general mass matrix (1) with only two parameters η and ϵ , with an additional mass scale m_0 . The expression for the prediction on solar mixing angle (2) will now depend only on the ratio of these two variables, η and ϵ . Such consideration in the reduction of the number of parameters in the texture, gives a firm handle on the phenomenological analysis of the mass matrices obeying μ - τ symmetry. The parametrisations presented below are not unique. We give such parametrisation for normal hierarchy as well as inverted hierarchy. The numerical analysis is performed using Mathematica.

3.1 Parametrisation for normal Hierarchy

We present here two forms of parametrisation related to the mass matrices in eqs.(1) and (12) discussed in section 2 and 2.1, with two parameters η and ϵ in the texture, and a common mass scale parameter m_0 .

Case (i) with $X \neq 0$: The general mass matrix of the form(1) with no zero texture, is parametrised by

$$m_{LL}(NH) = \begin{pmatrix} -\eta & -\epsilon & -\epsilon \\ -\epsilon & 1-\epsilon & -1 \\ -\epsilon & -1 & 1-\epsilon \end{pmatrix} m_0 \quad (17)$$

This predicts an expression for solar mixing angle,

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{|\eta/\epsilon - 1|}$$

where the ratio $x = \frac{\eta}{\epsilon}$ is the “flavor twister” in this case. The possible solutions for lowering solar angle beyond tribimaximal solar mixing, are given by $|\eta/\epsilon - 1| \leq 1$ which leads to $\eta/\epsilon \leq 0$ and $\eta/\epsilon \geq 2$. The numerical predictions on deviation from tri-bimaximal mixings for the case $\eta/\epsilon \leq 0$ are presented in Table-1. The predictions on Δm_{23}^2 and Δm_{21}^2 are consistent with observed data for a wide range of solar angle $\tan^2 \theta_{12} = (0.50 - 0.35)$.

As an example we cite a representative case for $\tan^2 \theta_{12} = 0.45$. Taking input values for $\eta/\epsilon = -0.1595$, $\epsilon = 0.1894$ and $m_0 = 0.029eV$, we get the three mass eigenvalues, $m_i = (0.00609, -0.0107, 0.05251)eV$, leading to $\Delta m_{21}^2 = 7.75 \times 10^{-5}eV^2$ and $\Delta m_{23}^2 = 2.64 \times 10^{-3}eV^2$.

Case (ii) with $X = 0$: Following the procedure in eq.(12), the mass matrix (17) is now modified as

$$m_{LL}(NH) = \begin{pmatrix} 0 & -\epsilon & -\epsilon \\ -\epsilon & 1-\epsilon & -1+\eta \\ -\epsilon & -1+\eta & 1-\epsilon \end{pmatrix} m_0 \quad (18)$$

where the solar mixing angle prediction corresponding to a choice of flavor twister, is same as that of eq.(17), except that the relation $\tan^2 \theta_{12} = \frac{m_1}{m_2}$ is valid in this case since 1-1 term in the mass matrix is zero. Table-2 gives the numerical predictions at lower values of solar angle. The predictions on solar and atmospheric mass scales are consistent with the recent experimental data. In order to have the result for $\tan^2 \theta_{12} = 0.45$, we take the input values for $\eta/\epsilon = -0.1595$, $\epsilon = 0.17$ and $m_0 = 0.028eV$, and we get $m_i = (0.00452, -0.01004, 0.05199)eV$ leading to $\Delta m_{21}^2 = 8.03 \times 10^{-5}eV^2$ and $\Delta m_{23}^2 = 2.60 \times 10^{-3}eV^2$ respectively.

3.2 Parametrisation of inverted hierarchy

We again consider two cases with $m_3 \neq 0$ and $m_3 = 0$ respectively for inverted hierarchical model, with only two parameters η and ϵ , in addition to an overall mass scale m_0 . As discussed earlier, we do not expect zero texture in 1-1 element in inverted hierarchical model.

Case (i) with $m_3 \neq 0$: A suitable parametrisation for mass matrix (1) has the form[13],

$$m_{LL}(IH) = \begin{pmatrix} 1 - 2\epsilon & -\epsilon & -\epsilon \\ -\epsilon & 1/2 & 1/2 - \eta \\ -\epsilon & 1/2 - \eta & 1/2 \end{pmatrix} m_0. \quad (19)$$

This gives the prediction of solar angle,

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{|(2 - \eta/\epsilon)|}.$$

This leads to the condition for lowering solar mixing angle beyond tribimaximal mixing, $|2 - \eta/\epsilon| \leq 1$ which has two possibilities[13]: $\eta/\epsilon \leq 1$ and $\eta/\epsilon \geq 3$. The corresponding numerical predictions are given in Table-3 for three types: A, B, and C. Type A means $m_i = (m_1, m_2, m_3)$, type B means $m_i = (m_1, -m_2, m_3)$ and type C means $m_i = (m_1, m_2, -m_3)$ respectively, which are related to CP phases. The result shows that these types have a common mass matrix and hence a common origin.

For a demonstration, we cite numerical results for representative cases for $\tan^2 \theta_{12} = 0.45$. Type A: Taking input values for $\eta/\epsilon = 0.8405$, $\eta = 0.00465$, we get $m_i = (0.04918, 0.05003, 0.00023)eV$ leading to $\Delta m_{21}^2 = 8.39 \times 10^{-5} eV^2$ and $\Delta m_{23}^2 = 2.50 \times 10^{-3} eV^2$.

Type B: With input values $\eta/\epsilon = 0.8405$, $\eta = 0.58715$, we get $m_i = (-0.05299, 0.05378, 0.02936)eV$ leading to $\Delta m_{21}^2 = 8.38 \times 10^{-5} eV^2$ and $\Delta m_{23}^2 = 2.03 \times 10^{-3} eV^2$.

Type C: For input values for $\eta/\epsilon = 3.16$, $\eta = -0.017$, we have $m_i = (0.05028, 0.05111, -0.00085)eV$ leading to $\Delta m_{21}^2 = 8.34 \times 10^{-5} eV^2$ and $\Delta m_{23}^2 = 2.61 \times 10^{-3} eV^2$. In all three cases we take $m_0 = 0.05eV$ as input.

Case (ii) with $m_3 = 0$: Following eq.(14), we express eq.(19) in the following form of mass matrix,

$$m_{LL}(IH) = \begin{pmatrix} 1 - 2\epsilon & -\epsilon & -\epsilon \\ -\epsilon & 1/2 - \eta/2 & 1/2 - \eta/2 \\ -\epsilon & 1/2 - \eta/2 & 1/2 - \eta/2 \end{pmatrix} m_0. \quad (20)$$

The numerical predictions are given in Table-4 (for type B), Table-5 (for type A) and Table-6 (for type C) for the range of solar angle, $\tan^2 \theta_{12} = (0.5 - 0.35)$.

We again give corresponding results for $\tan^2 \theta_{12} = 0.45$ in three types.

Type A: Taking input values for $\eta/\epsilon = 0.8405$, $\eta = 0.00445$ and $m_0 = 0.05eV$, we get $m_i = (0.04922, 0.05003, 0.0)eV$ leading to $\Delta m_{21}^2 = 8.03 \times 10^{-5}eV^2$ and $\Delta m_{23}^2 = 2.50 \times 10^{-3}eV^2$.

Type B: With input values $\eta/\epsilon = 0.8405$, $\eta = 0.58695$ and $m_0 = 0.048eV$, we get $m_i = (-0.05084, 0.05163, 0.0)eV$ leading to $\Delta m_{21}^2 = 8.06 \times 10^{-5}eV^2$ and $\Delta m_{23}^2 = 2.58 \times 10^{-3}eV^2$.

Type C: For input values for $\eta/\epsilon = 3.16$, $\eta = -0.01645$ and $m_0 = 0.05eV$, we have $m_i = (0.05027, 0.05107, 0.0)eV$ leading to $\Delta m_{21}^2 = 8.06 \times 10^{-5}eV^2$ and $\Delta m_{23}^2 = 2.61 \times 10^{-3}eV^2$.

3.3 Understanding the parametrisation in neutrino mass matrices

A: Inverted hierarchy: In order to understand the form of mass matrix parametrised in eq.(19), we start with two parts[24] of neutrino mass matrix, $m_{LL} = m_{LL}^o + \Delta m_{LL}$, which can be diagonalised by tri-bimaximal mixing matrix (5). For the inverted hierarchy the structure of the dominant term m_{LL}^o having μ - τ symmetry, is given by

$$m_{LL}^o = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0. \quad (21)$$

which is diagonalised as

$$O_{23}^T m_{LL}^o O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} m_0 \quad (22)$$

The second perturbative term Δm_{LL} can also be diagonalised by $(O_{23}O_{12})$,

$$\Delta m_{LL} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} m_0(-\eta) \quad (23)$$

where η is a very small parameter. The diagonalisation with tri-bimaximal mixing matrix (5),

$$(O_{23}O_{12})^T \Delta m_{LL} (O_{23}O_{12}) = O_{12}^T (O_{23}^T \Delta m_{LL} O_{23}) O_{12}$$

gives

$$O_{12}^T O_{23}^T \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} O_{23} O_{12} m_0(-\eta) \quad (24)$$

$$= O_{12}^T \begin{pmatrix} 2 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} O_{12} m_0(-\eta) \quad (25)$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} m_0(-\eta). \quad (26)$$

Thus, from eqs.(22) and (26), the diagonalisation of the total mass matrix,

$$U_{TBM}^T m_{LL} U_{TBM} = O_{23}^T m_{LL}^o O_{23} + (O_{23}O_{12})^T \Delta m_{LL} (O_{23}O_{12})$$

leads to

$$= \begin{pmatrix} 1-3\eta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \eta \end{pmatrix} m_0. \quad (27)$$

The deviation of solar angle from tri-bimaximal mixings can be introduced through the replacement Δm_{LL} by $\Delta m'_{LL}$ using a flavor twister $x = \frac{\epsilon}{\eta}$ where

$$\Delta m'_{LL} = \begin{pmatrix} 2x & x & x \\ x & 0 & 1 \\ x & 1 & 0 \end{pmatrix} m_0(-\eta) \quad (28)$$

which still has μ - τ symmetry. This can be diagonalised by O_{23} but O_{12} is now replaced by a new matrix O'_{12} . Thus $O_{12}^T (O_{23}^T \Delta m'_{LL} O_{23}) O'_{12}$ leads to

$$O_{12}^T O_{23}^T \begin{pmatrix} 2x & x & x \\ x & 0 & 1 \\ x & 1 & 0 \end{pmatrix} O_{23} O'_{12} m_0(-\eta) \quad (29)$$

$$= O_{12}^T \begin{pmatrix} 2x & 2x/\sqrt{2} & 0 \\ 2x/\sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} O'_{12} m_0(-\eta) \quad (30)$$

$$= \begin{pmatrix} (1+2x+y)/2 & 0 & 0 \\ 0 & (1+2x-y)/2 & 0 \\ 0 & 0 & -1 \end{pmatrix} m_0(-\eta) \quad (31)$$

where $y = \sqrt{1 - 4x + 12x^2}$. The new solar angle calculated from O'_{12} , is now given by

$$\tan 2\theta_{12} = \left| \frac{2\sqrt{2}}{2 - 1/x} \right|$$

The corresponding new mass eigenvalues for m'_{LL} are calculated as

$$m_{1,2} = \frac{m_o}{2} [2 - \eta(1 + 2x \pm y)]; m_3 = \eta m_o \quad (32)$$

After identification of $x = \epsilon/\eta$, we obtain the same mass matrix(19).

B.Normal Hierarchy:

In case of normal hierarchy(17), we start with two parts[24] of neutrino mass matrix, $m_{LL} = m_{LL}^o + \Delta m_{LL}$, which can be diagonalised by tribimaximal mixing matrix(5). The structure of the dominant term m_{LL}^o having μ - τ symmetry, can be taken as

$$m_{LL}^o = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} m_0 \quad (33)$$

which can be diagonalised by

$$O_{23}^T m_{LL}^o O_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} m_0. \quad (34)$$

However, the second term Δm_{LL} which can be diagonalised by $(O_{23}O_{12})$, can be taken as

$$\Delta m_{LL} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} m_0(-\epsilon) \quad (35)$$

where ϵ is a very small real parameter. The diagonalisation of eq.(35) with tribimaximal mixing matrix,

$$(O_{23}O_{12})^T \Delta m_{LL} (O_{23}O_{12}) = O_{12}^T (O_{23}^T \Delta m_{LL} O_{23}) O_{12}$$

leads to

$$O_{12}^T O_{23}^T \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} O_{23} O_{12} m_0(-\epsilon) \quad (36)$$

$$= O_{12}^T \begin{pmatrix} 2 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} O_{12} m_0(-\epsilon) \quad (37)$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0(-\epsilon). \quad (38)$$

From eqs.(34) and (38), the diagonalisation,

$$U_{TBM}^T m_{LL} U_{TBM} = O_{23}^T m_{LL}^o O_{23} + (O_{23} O_{12})^T \Delta m_{LL} (O_{23} O_{12}),$$

leads to

$$= \begin{pmatrix} -3\epsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (2-\epsilon) \end{pmatrix} m_0. \quad (39)$$

The deviation of solar angle from tribimaximal mixings can be done through the replacement Δm_{LL} by $\Delta m'_{LL}$ using a flavor twister $x = \frac{\eta}{2\epsilon}$,

$$\Delta m'_{LL} = \begin{pmatrix} 2x & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} m_0(-\epsilon). \quad (40)$$

which still obeys μ - τ symmetry and can be diagonalised by O_{23} . Thus

$$O_{12}^T (O_{23}^T \Delta m'_{LL} O_{23}) O_{12}'$$

leads to

$$O_{12}'^T O_{23}^T \begin{pmatrix} 2x & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} O_{23} O_{12}' m_0(-\epsilon) \quad (41)$$

$$= O_{12}'^T \begin{pmatrix} 2x & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} O_{12}' m_0(-\epsilon) \quad (42)$$

$\tan^2 \theta_{12}$	η/ϵ	range of ϵ	$\Delta m_{21}^2 (10^{-5} eV^2)$	$\Delta m_{23}^2 (10^{-3} eV^2)$
0.500	0.0	0.1640 – 0.1880	6.77 – 8.83	2.75 – 2.64
0.450	-0.1595	0.1762 – 0.2025	6.71 – 8.86	2.70 – 2.59
0.382	-0.4142	0.2080 – 0.2380	6.74 – 8.82	2.57 – 2.45
0.350	-0.5538	0.2356 – 0.2707	6.72 – 8.87	2.46 – 2.31

Table 1: Normal hierarchy with non-zero 1-1 term ($X \neq 0$) in the texture. Input value of $m_0 = 0.029 eV$.

$\tan^2 \theta_{12}$	η/ϵ	range of ϵ	$\Delta m_{21}^2 (10^{-5} eV^2)$	$\Delta m_{23}^2 (10^{-3} eV^2)$
0.50	0.0	0.175 – 0.195	7.20 – 8.94	2.52 – 2.44
0.45	-0.1595	0.160 – 0.180	7.11 – 9.00	2.64 – 2.57
0.35	-0.5538	0.135 – 0.150	7.16 – 8.85	2.87 – 2.83

Table 2: Normal hierarchy with zero 1-1 term ($X = 0$) in the texture and $m_0 = 0.028 eV$

$$= \begin{pmatrix} (1+2x-y)/2 & 0 & 0 \\ 0 & (1+2x+y)/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0(-\epsilon) \quad (43)$$

where $y = \sqrt{9 - 4x + 4x^2}$ and new O'_{12} can be obtained in principle. The new solar mixing angle is given by

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{2x-1}$$

The corresponding new mass eigenvalues for m'_{LL} are

$$m_{1,2} = \frac{m_o}{2}[-\epsilon \pm \eta \pm \epsilon y]; m_3 = (2 - \epsilon)m_o. \quad (44)$$

After substitution of $x = \eta/(2\epsilon)$, we recover the earlier mass matrix in eq.(17). For the value $x = 0$, we have the tribimaximal condition $O_{12} = O'_{12}$ leading to $\tan^2 \theta_{12} = 0.5$.

$Type$	$\tan^2 \theta_{12}$	η/ϵ	range of η	$\Delta m_{21}^2 (10^{-5} eV^2)$	$\Delta m_{23}^2 (10^{-3} eV^2)$
A	0.50	1.0	0.0048 – 0.0064	7.15 – 9.51	2.50 – 2.50
B	0.50	1.0	0.6607 – 0.6618	9.50 – 7.20	1.41 – 1.41
C	0.50	3.0	–0.0187 to –0.0142	9.41 – 7.20	2.63 – 2.60
A	0.45	0.8405	0.0040 – 0.0053	7.29 – 9.54	2.5 – 2.5
B	0.45	0.8405	0.5865 – 0.5878	9.52 – 7.27	2.03 – 2.03
C	0.45	3.1600	–0.0193 to –0.0147	9.47 – 7.24	2.63 – 2.60
A	0.35	0.4462	0.0020 – 0.0026	7.22 – 9.30	2.51 – 2.51
B	0.35	0.4462	0.3622 – 0.3628	9.43 – 7.22	4.01 – 4.01
C	0.35	3.5500	–0.0206 to –0.0157	9.50 – 7.20	2.63 – 2.60

Table 3: Inverted hierarchy with $m_3 \neq 0$ for Type A, Type B, and Type C, explained in the text. Input value of $m_0 = 0.05 eV$.

$\tan^2 \theta_{12}$	η/ϵ	range of η	$\Delta m_{21}^2 (10^{-5} eV^2)$	$\Delta m_{23}^2 (10^{-3} eV^2)$
0.500	1.0	0.6601 – 0.6615	8.99 – 7.09	2.304 – 2.304
0.450	0.8405	0.5864 – 0.5875	8.96 – 7.15	2.665 – 2.666
0.382	0.5858	0.4495 – 0.4502	8.86 – 7.14	3.436 – 3.438
0.350	0.4462	0.3621 – 0.3627	8.97 – 6.99	3.995 – 3.999

Table 4: Inverted hierarchy with Type B: $m_i = (m_1, -m_2, 0)$. Input value $m_0 = 0.048 eV$.

$\tan^2 \theta_{12}$	η/ϵ	range of η	$\Delta m_{21}^2 (10^{-5} eV^2)$	$\Delta m_{23}^2 (10^{-3} eV^2)$
0.50	1.0000	0.0048 – 0.0060	7.15 – 8.92	2.500 – 2.500
0.45	0.8405	0.0039 – 0.0050	7.05 – 9.02	2.503 – 2.503
0.35	0.4462	0.0020 – 0.0025	7.20 – 8.98	2.510 – 2.510

Table 5: Inverted hierarchy with Type A: $m_i = (m_1, m_2, 0)$, $m_0 = 0.05 eV$

$\tan^2 \theta_{12}$	η/ϵ	range of η	$\Delta m_{21}^2 (10^{-5} eV^2)$	$\Delta m_{23}^2 (10^{-3} eV^2)$
0.50	3.00	–0.0176 to –0.0142	8.93 – 7.18	2.610 – 2.596
0.45	3.16	–0.0182 to –0.0147	8.93 – 7.20	2.610 – 2.596
0.35	3.55	–0.0195 to –0.0157	8.99 – 7.22	2.622 – 2.598

Table 6: Inverted hierarchy with Type C: $m_i = (m_1, m_2, 0)$, $m_0 = 0.05 eV$

4 Effects of renormalisation group analysis in MSSM for large $\tan \beta$

There are excellent papers[3,23, 25,26] devoted to radiative corrections on neutrino masses and mixings, and on Quark-Lepton Complementarity relation. However the problem with inverted hierarchy with opposite CP-parities in the first two mass eigenvalues (type B), is not yet settled completely. In particular, it has been shown with analytic calculations in Ref.[23] that for large $\tan \beta = 50$ the radiatively generated low-scale value of Δm_{21}^2 has a negative sign and this contradicts the experimental data. We are interested here to examine this conjecture for high-scale input value of solar angle given by tribimaximal mixing and below. The effect of non-zero value of m_3 on the evolution of mixing angles as well as Δm_{21}^2 will be investigated in greater details. We take high-scale input values of $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ in the numerical analysis.

We start with a very brief outline on the procedure of RG analysis without phases, while referring for details to our earlier works[27,28]. The effects of quantum radiative corrections of neutrino masses and mixings in MSSM, lead to the low-energy neutrino mass matrix,

$$m_{LL}(t_0) \sim \begin{pmatrix} X(t_u) & Y(t_u) & Y(t_u)e^{-I_\tau} \\ Y(t_u) & Z(t_u) & W(t_u)e^{-I_\tau} \\ Y(t_u)e^{-I_\tau} & W(t_u)e^{-I_\tau} & Z(t_u)e^{-2I_\tau} \end{pmatrix} R_0 \quad (45)$$

where

$$R_0 = \exp[(6/5)I_{g_1} + 6I_{g_2} - 6I_{top}],$$

$$I_{g_i} = \frac{1}{16\pi^2} \int_{t_0}^{t_u} g_i^2(t) dt, i = 1, 2, 3,$$

$$I_{h_f} = \frac{1}{16\pi^2} \int_{t_0}^{t_u} h_f^2(t) dt, f = top, b, \tau,$$

$$t = \ln(\mu/1GeV), t_0 = \ln(m_t/1GeV), t_u = \ln(M_U/1GeV).$$

The above analytic solution of the neutrino mass matrix at low-energy scale is possible only where charged lepton mass matrix is diagonal. We have also neglected h_e^2, h_μ^2 compared to h_τ^2 , and for large $\tan \beta \sim 55 - 60$, we can take $R_0 \sim 1$ and $c = e^{I_\tau} \sim 1.06$. Thus the low-energy mass matrix (45) has the

form,

$$m_{LL}(t_0) \sim \begin{pmatrix} X & Y & Y/c \\ Y & Z & W/c \\ Y/c & W/c & Z/c^2 \end{pmatrix} R_0. \quad (46)$$

In this approach the neutrino mass matrix evolves as a whole from high scale to low scale, and diagonalisation of the mass matrix at any particular energy scale leads to the physical neutrino mass eigenvalues as well as mixing angles. This approach is numerically consistent with other approach where neutrino mass eigenvalues and three mixings angles evolve separately through coupled RG equations. In MSSM we have the following RG equations[29],

$$\frac{d}{dt}m_i = \frac{1}{16\pi^2} \left[\left(-\frac{6}{5}g_1^2 - g_2^2 + 6h_t^2 \right) + 2h_\tau^2 U_{\tau i}^2 \right] m_i, i = 1, 2, 3, \quad (47)$$

$$\frac{ds_{12}}{dt} = \frac{1}{16\pi^2} h_\tau^2 c_{12} [c_{23}s_{13}s_{12}U_{\tau 1}A_{31} - c_{23}s_{13}c_{13}U_{\tau 2}A_{32} + U_{\tau 1}U_{\tau 2}A_{21}], \quad (48)$$

$$\frac{ds_{13}}{dt} = \frac{1}{16\pi^2} h_\tau^2 c_{23} c_{13}^2 [c_{12}U_{\tau 1}A_{31} + s_{12}U_{\tau 2}A_{32}], \quad (49)$$

$$\frac{ds_{23}}{dt} = \frac{1}{16\pi^2} h_\tau^2 c_{23}^2 [-s_{12}U_{\tau 1}A_{31} + c_{12}U_{\tau 2}A_{32}] \quad (50)$$

where $A_{ki} = \frac{m_k + m_i}{m_k - m_i}$ and U_{fi} are the elements in MNS matrix(3) parametrised by (neglecting CP Dirac phase),

$$U_{MNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - c_{23}s_{13}s_{12} & c_{13}c_{23} \end{pmatrix} \quad (51)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ respectively.

We follow the standard procedure for a complete numerical analysis of the RGEs for neutrino masses and mixing angles in two consecutive steps (i) bottom-up running in the first place where running of third family Yukawa couplings and three gauge couplings in MSSM, are carried out from top-quark mass scale at low energy end to high energy scale[30]. In the present analysis we consider the high scale value as the unification scale, $M_U = 1.6 \times 10^{16} GeV$, with large $\tan \beta = 60$ as input value. For simplicity of the calculation we take approximately the SUSY breaking scale at the top-quark mass scale $t_0 = \ln(m_t)$. We adopt the standard procedure to get the values of gauge couplings at top-quark mass scale from experimental data, using

one-loop RGEs, assuming the existence of one-light Higgs doublet and five quark flavors below top-quark scale. The values of three Yukawa couplings and three gauge couplings are calculated at high unification scale. (ii) In the second top-down approach, the runnings of three neutrino masses and three mixing angles are carried out simultaneously with the running of Yukawa and gauge couplings, from high to low scale, using the input values of Yukawa and gauge couplings evaluated in the first stage of running[28].

The normal hierarchical model is almost stable under radiative correction[3,4] and is of little interest. This is evident from the fact that the 1-1 term in the mass matrix is almost zero. There is a mild increase in both solar and atmospheric mixing angles while running from high to low scale. The mass splitting is found to be acceptable and we are not repeating the same investigation here.

The evolution of neutrino masses with energy scale in case of inverted hierarchical model with opposite CP-parities, is highly affected with the high scale input value of the solar mixing angle. We observe that the model is not stable for input value of solar angle below $\theta_{12} = 37^\circ$ at large $\tan\beta$ values. We summarise the following points:

In Fig.1 we show the level crossing of the magnitudes of two mass eigenvalues $|m_1|$ and $|m_2|$ at around $\theta_{21} = 35.24^\circ$, leading to a negative value for Δm_{21}^2 at low energy scale. This fact is shown in Fig.2 for three different high scale input values of solar mixing angles. For higher value $\tan^2\theta_{12} = 0.8$, low energy value of Δm_{21}^2 falls in the positive range, but for $\tan^2\theta_{12} = 0.5$ and $\tan^2\theta_{12} = 0.4$, it falls in the negative range due to level crossing of first two mass eigenvalues. The sensitivity of the low energy Δm_{21}^2 with high scale input solar mixing angle is shown in Fig.3. In the analysis we take high scale input values: $m_1 = -0.04918eV$, $m_2 = 0.05eV$, $m_3 = 0.03306eV$, $\sin\theta_{23} = 0.70711$ and $\sin\theta_{13} = 0$ respectively.

The effect of non-zero input value of m_3 at high scale, is important for maintaining the stability of Δm_{21}^2 . Even for larger solar angle $\tan\theta_{12} = 0.8$, case with $m_3 = 0.033eV$ gives better result than the case with $m_3 = 0$, and this point is demonstrated in Fig.5. This definitely lies outside the observed data. However, zero value of m_3 is useful for stability of the evolution for atmospheric angle. Fig.4 depicts the evolution of $\sin\theta_{23}$ (upper pair) for two values of $m_3 = 0$ and $m_3 = 0.033eV$, respectively. In case of solar angle the effect of m_3 is negligible.

In short, the inverted hierarchical model with opposite CP-parities, is not so stable under RG running in MSSM for larger $\tan\beta$ region where the effect

of RG is maximum in Δm_{21}^2 and $\sin \theta_{23}$. At low values of $\tan \beta$, the RG effects are normally small.

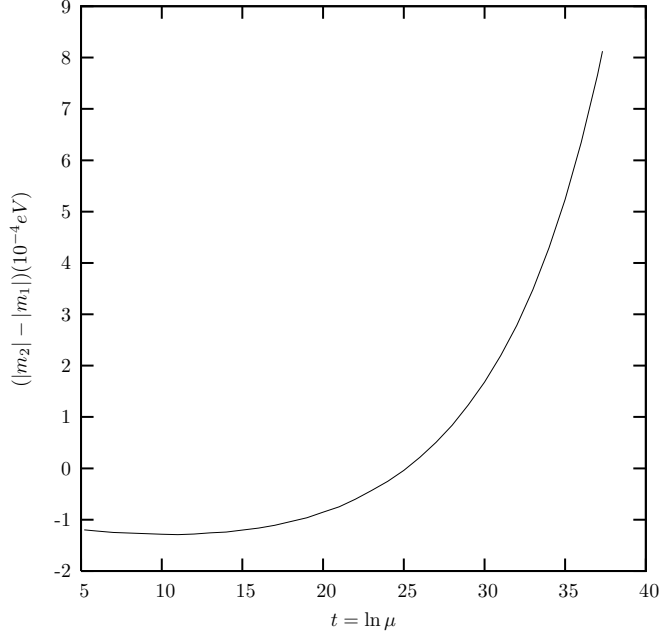


Fig.1 Evolution of the difference of the magnitudes of first two neutrino mass eigenvalues $|m_1|$ and $|m_2|$, with energy scale $\ln(\mu/1\text{GeV})$ in inverted hierarchy. At lower energy scale there is the level crossing which makes it negative. High scale input values are $m_1 = -0.04919\text{eV}$, $m_2 = 0.05\text{eV}$, $m_3 = 0.03306\text{eV}$, $\sin \theta_{23} = 0.70711$, $\sin \theta_{12} = 0.57735$, $\sin \theta_{13} = 0$ respectively.

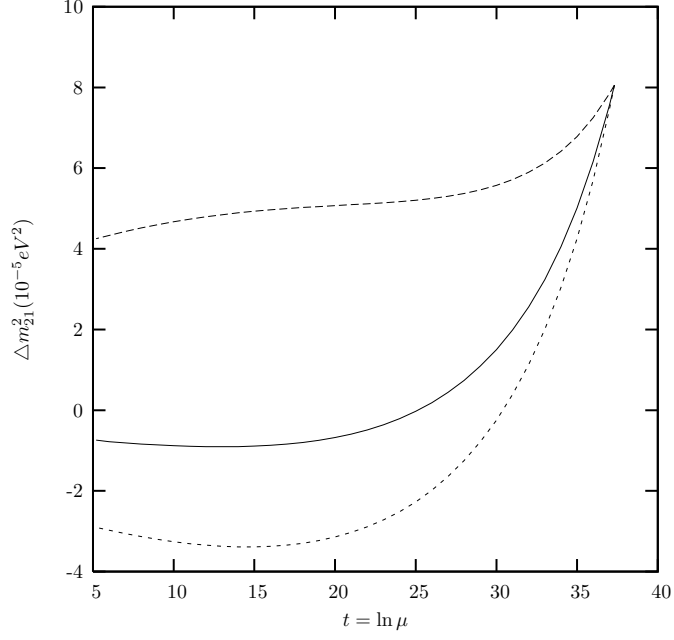


Fig.2 Evolution of Δm_{21}^2 with energy scale $\ln(\mu/1\text{GeV})$ for different high scale input values of solar angle (from top to bottom): $\tan^2 \theta_{12} = 0.8$ (dashed-line), $\tan^2 \theta_{12} = 0.5$ (solid line), $\tan^2 \theta_{12} = 0.4$ (dotted line). Other input parameters are same as those in Fig.1. Corresponding graphs for cases with $m_3 = 0$ condition, will be more severe (see also in Fig.5).

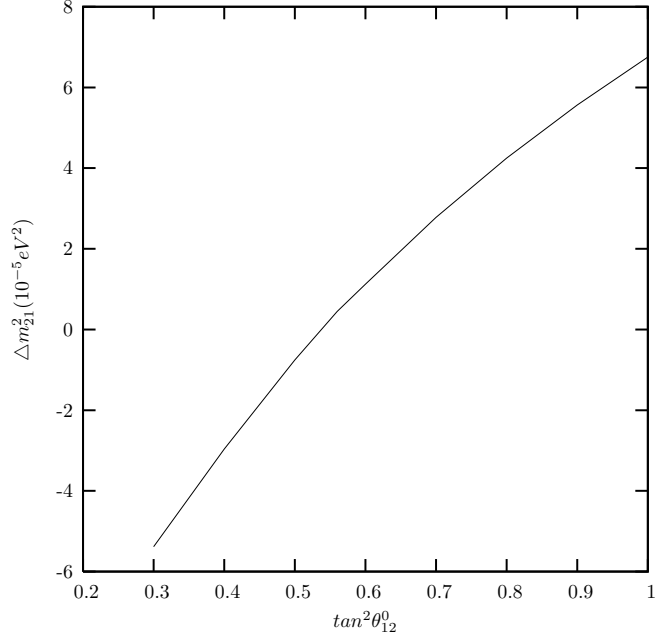


Fig.3 Sensitivity of low-energy values of Δm_{21}^2 versus different high scale input values of solar mixing angles: $\tan^2 \theta_{12}$, as analysed in Fig.2. Other input parameters are same as those in Fig.1.

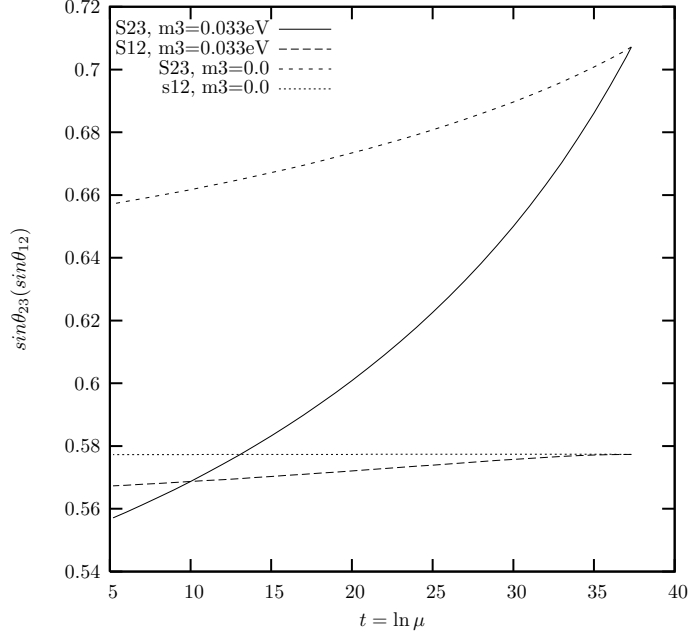


Fig.4 Evolution of $\sin \theta_{23}$ for $m_3 = 0$ and $m_3 = 0.033\text{eV}$ in the first two from top; and of $\sin \theta_{12}$ for $m_3 = 0$ and $m_3 = 0.033\text{eV}$ in the last two, respectively. Other input parameters are same as those in Fig.1.

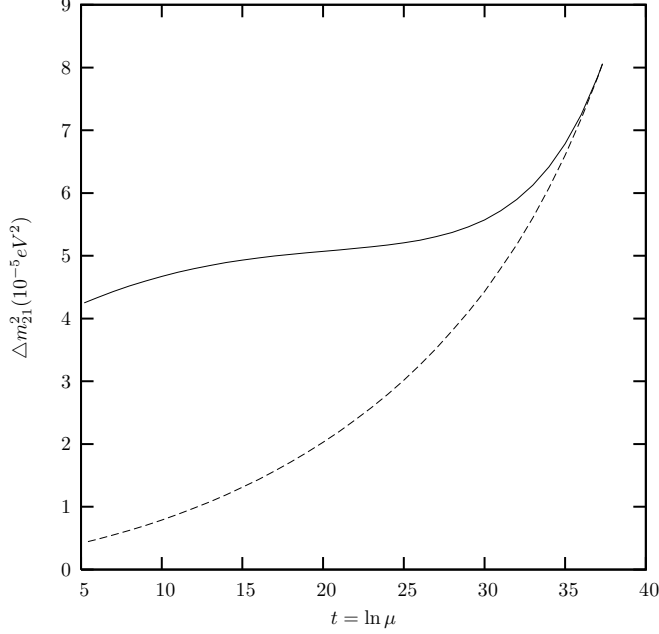


Fig.5 Evolution of Δm_{21}^2 with energy scale for two different values of m_3 (A: solid line for $m_3 = 0.033\text{eV}$ and B: dashed line for $m_3 = 0$) respectively. Case A is more stable than case B. Other high scale input values are same as those in Fig.1 except for solar mixing angle $\tan^2 \theta_{12} = 0.8$.

5 Estimation of the baryon asymmetry of the Universe

We present the baryon asymmetries calculated from these mass models m_{LL} discussed in section 3, using the inverse seesaw formula $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. In left-right symmetric models such as SO(10) GUT, the right-handed neutrino in addition to its role in seesaw mechanism, can also explain the baryon asymmetry of the Universe through leptogenesis[31]. The heavy RH neutrino being a Majorana neutrino, can have an asymmetric decay into lepton and an anti-lepton with different rates for the lepton and the anti-lepton, thereby generating a CP asymmetry which is one of the Shkarov's three conditions[32] required for generating the baryon asymmetry of the Universe.

$$\begin{aligned}
N_R &\rightarrow l_L + \phi^\dagger \\
&\rightarrow \bar{l}_L + \phi
\end{aligned}$$

where l_L is the lepton and \bar{l}_L is the antilepton[33]. For the temperature of the Universe less than the mass of the decaying lightest of the heavy RH neutrino, the out-of-equilibrium condition is reached as the inverse decay is blocked. The CP asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of lightest of heavy right-handed Majorana neutrino N_1 , is defined by[33,34] for standard model (SM) case,

$$\epsilon_1 = \frac{3}{16\pi} \left[\frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_2} + \frac{\text{Im}[(h^\dagger h)_{13}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_3} \right] \quad (52)$$

where $h = m'_{LR}/v$ is the Yukawa coupling of the Dirac neutrino mass matrix defined in the basis where the right-handed neutrino mass matrix is diagonal. Here M_1, M_2, M_3 are the physical right-handed Majorana masses taken in hierarchical order ($M_1 < M_2 < M_3$).

For quasi-degenerate spectrum i.e., for $M_1 \simeq M_2 < M_3$, the asymmetry is largely enhanced by a resonance factor[16,34,35]. In such situation, the lepton asymmetry is modified[34,35,36] to

$$\epsilon_1 = \frac{1}{8\pi} \frac{\text{Im}[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} R \quad (53)$$

where

$$R = \frac{M_2^2(M_2^2 - M_1^2)}{(M_1^2 - M_2^2)^2 + \Gamma_2^2 M_1^2} \text{ and } \Gamma_2 = \frac{(h^\dagger h)_{22} M_2}{8\pi}$$

Again, the lepton asymmetry is further converted into baryon asymmetry via a non-perturbative sphaleron process[37]. The ratio of baryon asymmetry to entropy Y_B is related to the lepton asymmetry through the relation, $Y_B = w Y_{B-L} = \frac{w}{w-1} Y_L$ where $w = (8N_F + 4N_H)/(22N_F + 13N_H)$. The baryon asymmetry of the Universe Y_B is defined as the ratio of the baryon number density η_B to the photon density η_γ where $s = 7.04n_\gamma$. This can be compared with the observational data $Y_B = (6.21 \pm 0.160 \times 10^{-10})$. In SM it can be expressed as

$$Y_B^{SM} \simeq d\kappa_1 \epsilon_1 \quad (54)$$

where $d = 7.04 \times \frac{1}{g_i^*} \frac{w}{w-1}$. For SM with $N_F = 3, N_H = 1, g_i^* = 106.75$, we have $d \simeq 3.62 \times 10^{-2}$. This value[38] can be compared with lower value $d \simeq 0.98 \times 10^{-2}$ used by other authors[7,17,39].

In the expression for baryon-to-photon ratio the efficiency factor (also known as dilution factor) κ_1 describes the washout factor of the lepton asymmetry due to various lepton number violating processes. This factor mainly depends on the effective neutrino mass \tilde{m}_1 defined by

$$\tilde{m}_1 = \frac{(h^\dagger h)_{11} v^2}{M_1}$$

where v is the vev of the standard model Higgs field $v = 174$ GeV.

For $10^{-2} eV < \tilde{m}_1 < 10^3 eV$, the washout factor κ_1 can be well approximated by[40,41]

$$\kappa_1(\tilde{m}_1) = 0.3 \left[\frac{10^{-3} eV}{\tilde{m}_1} \right] \left[\log \frac{\tilde{m}_1}{10^{-3}} \right]^{-0.6} \quad (55)$$

For numerical estimation of the baryon asymmetry we start with the given light Majorana neutrino mass matrix m_{LL} having $\mu - \tau$ symmetry given in section 3 and then translate this mass matrix to M_{RR} via inversion of the seesaw formula $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$ where m_{LR} is the Dirac neutrino mass matrix and M_{RR} is the right-handed Majorana neutrino mass matrix. For simplicity we consider the diagonal form of m_{LR} , and as also for three different possible choices of m_{LR} namely (i) up-quark mass matrix $(m, n) = (8, 4)$, (ii) charged-lepton mass matrix $(m, n) = (6, 2)$ and (iii) down-quark mass matrix $(m, n) = (4, 2)$, we choose a basis U_R where $M_{RR}^{diag} = U_R^T M_{RR} U_R = \text{diag}(M_1, M_2, M_3)$ with real and positive eigenvalues[39]. We transform $m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1)v$ to the U_R basis $m_{LR} \rightarrow m'_{LR} = m_{LR} U_R Q$, where

$$Q = \text{diag}(1, e^{i\alpha}, e^{i\beta})$$

is the arbitrary Majorana phases responsible for CP violation. It can be emphasised here that the origin of Majorana phases comes from M_{RR} . Here $v = 174$ GeV is the electroweak vev and $\lambda \simeq 0.3$ is the Wolfenstein parameter. For non-zero CP asymmetry the Majorana phases (α, β) are different from 0 and $\pi/2$. For diagonal Dirac mass matrices, the introduction of complex Majorana phases in MNS mixing matrix and in the diagonalising matrix

of right-handed Majorana mass matrix, have the same effect to get complex Dirac mass matrices entered in the expression of CP asymmetry. In this prime basis the Dirac neutrino Yukawa coupling becomes $h = \frac{m'_{LR}}{v}$. For demonstration we first consider the normal hierarchical model as an example given in eq.(17) for $(1, 1)$ term is not zero ($X \neq 0$).

The mass eigenvalues for RH Majorana neutrinos are $M_{RR}^{diag} = \text{diag}(3.93 \times 10^{11}, 4.09 \times 10^{11}, 2.87 \times 10^{14}) \text{GeV}$. For this structure we find $\text{Im}(h^\dagger h)_{11} = 7.5 \times 10^{-3}$; $\kappa_1 = 1.8 \times 10^{-4}$. The lepton asymmetry is found out to be 4.59×10^{-3} . Thus following above equations (52,53) we calculate Y_B^{SM} to be 3.77×10^{-8} taking m_{LR} to be charged lepton mass matrix (case (ii)). The m_{LL} has shown good prediction of correct neutrino mass parameters and mixing angles consistent with the recent data : $\Delta m_{21}^2 = 7.1 \times 10^{-5} \text{eV}^2$, $\Delta m_{23}^2 = 2.7 \times 10^{-3} \text{eV}^2$. The results for the other cases are given in the tables 7-16. In the calculation we consider the three possible choices (m, n) of diagonal Dirac mass matrices $m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1)v$. Throughout the calculation we fix the choice of values of the phases $(\alpha, \beta) = (\pi/4, \pi/4)$ as it gives the maximum numerical values of baryon asymmetry.

In tables 8,10 and 12, the predicted values of baryon asymmetry Y_B for inverted hierarchical neutrino mass models, lie on the lower side. Specially for Type A inverted hierarchy (IHA) we find the range $Y_B < 10^{-14}$ for $\tan^2 \theta_{12} = 0.50 - 0.35$. However Type B inverted hierarchy (IHB) leads to slightly better result $10^{-14} < Y_B < 10^{-9}$ for $\tan^2 \theta_{12} = 0.50$ but not so good for lower solar angles. There is still some room to accommodate the observed data but if we consider lower value of $d = 0.96 \times 10^{-2}$ then the model fails to give good results in both inverted hierarchical models.

From tables 14 and 16, the baryon asymmetry predictions $10^{-10} < Y_B < 10^{-8}$ from normal hierarchy (both models with $X = 0$ and $X \neq 0$ cases) are consistent with observed data for wide range of values of $\tan^2 \theta_{12} = 0.50 - 0.35$. These results can sustain even for lower value of $d = 0.96 \times 10^{-2}$ parameter.

It can be emphasised here that the present calculation of baryon asymmetry is based on the neutrino mass matrices obeying 2-3 symmetry for both tribimaximal mixings and deviations of solar mixing from it, and hence only Majorana phases are considered.

$\tan^2 \theta_{12}$	type	(m,n)	M_1	M_2	M_3
0.50	A	(4,2)	4.01×10^{10}	9.73×10^{12}	6.25×10^{16}
0.50	A	(6,2)	3.29×10^8	9.73×10^{12}	6.25×10^{16}
0.50	A	(8,4)	2.63×10^6	7.94×10^{10}	6.21×10^{16}
0.05	B	(4,2)	-1.91×10^{11}	2.71×10^{12}	5.59×10^{14}
0.05	B	(6,2)	-9.99×10^8	2.63×10^{12}	5.59×10^{14}
0.05	B	(8,4)	-8.09×10^6	2.13×10^{10}	5.57×10^{14}

Table 7: Inverted hierarchical models (types A,B) with $m_3 \neq 0$, with the predicted neutrino mass parameters, $\Delta m_{21}^2 = 7.29 \times 10^{-5} eV^2$ ($8.50 \times 10^{-5} eV^2$) and $\Delta m_{23}^2 = 2.49 \times 10^{-3} eV^2$ ($2.30 \times 10^{-3} eV^2$) for type A(B) respectively. The prediction of physical right-handed Majorana masses in GeV for different choices of Dirac neutrino mass matrices (m, n) .

$\tan^2 \theta_{12}$	type	(m,n)	$(h^\dagger h)_{11}$	k_1	ϵ_1	Y_B
0.50	A	(4,2)	6.65×10^{-5}	2.95×10^{-3}	1.92×10^{-10}	2.50×10^{-14}
0.50	A	(6,2)	5.31×10^{-7}	2.95×10^{-3}	1.56×10^{-12}	2.02×10^{-16}
0.50	A	(8,4)	4.30×10^{-9}	2.95×10^{-3}	1.26×10^{-14}	1.64×10^{-18}
0.50	B	(4,2)	5.62×10^{-4}	8.83×10^{-4}	3.05×10^{-5}	1.18×10^{-9}
0.50	B	(6,2)	5.01×10^{-6}	8.83×10^{-4}	2.69×10^{-7}	1.05×10^{-11}
0.50	B	(8,4)	4.06×10^{-8}	8.83×10^{-4}	2.18×10^{-9}	8.42×10^{-14}

Table 8: Predictions of ϵ_1 the CP asymmetry, Y_B the baryon asymmetry for inverted hierarchical models (A,B) in Table-7, with $m_3 \neq 0$, for $\tan^2 \theta_{12} = 0.50$.

$\tan^2 \theta_{12}$	type	(m,n)	M_1	M_2	M_3
0.45	A	(4,2)	4.01×10^{10}	9.73×10^{12}	6.59×10^{16}
0.45	A	(6,2)	3.25×10^8	9.73×10^{12}	6.59×10^{16}
0.50	A	(8,4)	2.63×10^6	7.94×10^{10}	6.54×10^{16}
0.45	B	(4,2)	-9.76×10^{10}	2.89×10^{12}	6.23×10^{14}
0.45	B	(6,2)	-8.01×10^8	2.82×10^{12}	6.23×10^{14}
0.45	B	(8,4)	-6.56×10^6	2.29×10^{10}	6.21×10^{14}

Table 9: Inverted hierarchical models (types A,B) with $m_3 \neq 0$ for $\tan^2 \theta_{12} = 0.45$ with the predicted neutrino mass parameters, $\Delta m_{21}^2 = 8.39 \times 10^{-5} eV^2$ ($8.30 \times 10^{-5} eV^2$) and $\Delta m_{23}^2 = 2.51 \times 10^{-3} eV^2$ ($2.90 \times 10^{-3} eV^2$) for type A(B) respectively. The predictions of physical right-handed Majorana neutrino masses in GeV for different choices of Dirac neutrino mass matrices (m, n) .

$\tan^2 \theta_{12}$	type	(m,n)	$(h^\dagger h)_{11}$	k_1	ϵ_1	Y_B
0.45	A	(4,2)	6.56×10^{-5}	2.90×10^{-3}	2.46×10^{-10}	3.20×10^{-14}
0.45	A	(6,2)	5.31×10^{-7}	2.90×10^{-3}	2.04×10^{-12}	2.59×10^{-16}
0.45	A	(8,4)	4.30×10^{-9}	2.90×10^{-3}	1.61×10^{-14}	2.09×10^{-18}
0.45	B	(4,2)	4.38×10^{-4}	8.83×10^{-4}	2.10×10^{-5}	8.66×10^{-10}
0.45	B	(6,2)	3.82×10^{-6}	8.83×10^{-4}	1.97×10^{-7}	7.64×10^{-12}
0.45	B	(8,4)	3.09×10^{-8}	8.83×10^{-4}	1.59×10^{-9}	6.18×10^{-14}

Table 10: Predictions of ϵ_1 the CP asymmetry, Y_B the baryon asymmetry for inverted hierarchical models (A,B) presented in Table-9 with $m_3 \neq 0$ for $\tan^2 \theta_{12} = 0.45$

$\tan^2 \theta_{12}$	type	(m,n)	M_1	M_2	M_3
0.35	A	(4,2)	4.01×10^{10}	9.73×10^{12}	1.39×10^{17}
0.35	A	(6,2)	3.25×10^8	9.73×10^{12}	1.39×10^{17}
0.35	A	(8,4)	2.63×10^6	7.94×10^{10}	1.37×10^{17}
0.35	B	(4,2)	-6.27×10^{10}	1.37×10^{12}	9.50×10^{14}
0.35	B	(6,2)	-5.15×10^8	3.14×10^{12}	9.50×10^{14}
0.35	B	(8,4)	-4.17×10^6	2.55×10^{10}	9.45×10^{14}

Table 11: Inverted hierarchical models (types A,B) with $m_3 \neq 0$ for $\tan^2 \theta_{12} = 0.35$ with the predicted neutrino mass parameters, $\Delta m_{21}^2 = 7.90 \times 10^{-5} eV^2$ ($8.65 \times 10^{-5} eV^2$) and $\Delta m_{23}^2 = 2.50 \times 10^{-3} eV^2$ ($4.0 \times 10^{-3} eV^2$) for type A(B) respectively. The predictions of physical right-handed Majorana masses in GeV for different choices of Dirac mass matrices (m, n) .

$\tan^2 \theta_{12}$	type	(m,n)	$(h^\dagger h)_{11}$	k_1	ϵ_1	Y_B
0.35	A	(4,2)	6.56×10^{-5}	2.90×10^{-3}	4.20×10^{-10}	2.54×10^{-14}
0.35	A	(6,2)	5.31×10^{-7}	2.90×10^{-3}	1.58×10^{-12}	2.05×10^{-16}
0.35	A	(8,4)	4.30×10^{-9}	2.90×10^{-3}	2.82×10^{-14}	1.66×10^{-18}
0.35	B	(4,2)	2.76×10^{-4}	8.83×10^{-4}	1.26×10^{-5}	4.88×10^{-10}
0.35	B	(6,2)	2.32×10^{-6}	8.83×10^{-4}	1.08×10^{-7}	4.19×10^{-12}
0.35	B	(8,4)	1.88×10^{-8}	8.83×10^{-4}	8.73×10^{-10}	3.39×10^{-14}

Table 12: Predictions of ϵ_1 the CP asymmetry, Y_B the baryon asymmetry for inverted hierarchical models (A,B) presented in Table-11 with $m_3 \neq 0$ for $\tan^2 \theta_{12} = 0.35$.

$\tan^2 \theta_{12}$	(m,n)	M_1	M_2	M_3
0.50	(4,2)	3.59×10^{12}	-5.48×10^{12}	2.89×10^{14}
0.50	(6,2)	3.93×10^{11}	-4.09×10^{11}	2.87×10^{14}
0.50	(8,4)	3.19×10^9	-3.22×10^9	2.85×10^{14}
0.45	(4,2)	1.83×10^{12}	-2.64×10^{13}	9.79×10^{13}
0.45	(6,2)	1.93×10^{10}	-2.11×10^{13}	9.43×10^{13}
0.45	(8,4)	1.56×10^8	-2.21×10^{11}	7.28×10^{13}
0.35	(4,2)	4.81×10^{11}	2.48×10^{13}	-1.80×10^{14}
0.35	(6,2)	4.01×10^9	2.43×10^{13}	-1.80×10^{14}
0.35	(8,4)	3.24×10^7	2.29×10^{14}	-1.55×10^{14}

Table 13: Predictions of M_1, M_2, M_3 in GeV for different (m, n) for Dirac mass matrices, in the normal hierarchical model for case $X = m(1, 1) \neq 0$. The predicted neutrino mass parameters are $\Delta m_{21}^2 (\Delta m_{23}^2) = 7.1 \times 10^{-5} eV^2 (2.1 \times 10^{-3} eV^2), 6.68 \times 10^{-5} eV^2 (2.67 \times 10^{-3} eV^2), 6.9 \times 10^{-5} eV^2 (2.4 \times 10^{-3} eV^2)$ for $\tan^2 \theta_{12} = 0.50, 0.45, 0.35$ respectively.

$\tan^2 \theta_{12}$	(m,n)	\tilde{m}	$(h^\dagger h)_{11}$	κ_1	ϵ_1	Y_B
0.50	(4,2)	4.3×10^{-11}	5.1×10^{-3}	3.4×10^{-3}	4.50×10^{-4}	6.75×10^{-8}
0.50	(6,2)	5.8×10^{-10}	7.5×10^{-3}	1.8×10^{-4}	4.59×10^{-3}	3.77×10^{-8}
0.50	(8,4)	5.8×10^{-10}	6.1×10^{-5}	1.8×10^{-4}	3.62×10^{-5}	2.96×10^{-10}
0.45	(4,2)	4.1×10^{-11}	2.4×10^{-3}	3.7×10^{-3}	5.54×10^{-4}	9.03×10^{-8}
0.45	(6,2)	6.6×10^{-11}	4.2×10^{-5}	2.0×10^{-3}	8.63×10^{-6}	7.56×10^{-10}
0.45	(8,4)	6.6×10^{-11}	3.4×10^{-7}	2.0×10^{-3}	1.47×10^{-7}	1.28×10^{-11}
0.35	(4,2)	2.8×10^{-11}	4.5×10^{-4}	5.2×10^{-3}	8.85×10^{-5}	2.07×10^{-8}
0.35	(6,2)	3.0×10^{-11}	3.9×10^{-6}	5.2×10^{-3}	3.59×10^{-7}	7.93×10^{-11}
0.35	(8,4)	3.0×10^{-11}	3.2×10^{-8}	5.2×10^{-3}	8.99×10^{-9}	2.07×10^{-12}

Table 14: Predictions of ϵ_1 and Y_B for various values of $\tan^2 \theta_{12}$ for normal hierarchical model, Case(i) $X=m(1,1) \neq 0$ presented in Table-13 .

$\tan^2 \theta_{12}$	(m,n)	M_1	M_2	M_3
0.50	(4,2)	3.57×10^{12}	-5.29×10^{12}	3.01×10^{14}
0.50	(6,2)	3.85×10^{11}	-3.99×10^{11}	2.99×10^{14}
0.50	(8,4)	3.13×10^9	-3.25×10^9	2.97×10^{14}
0.45	(4,2)	3.72×10^{12}	-5.67×10^{12}	2.95×10^{14}
0.45	(6,2)	4.07×10^{11}	-4.23×10^{11}	2.93×10^{14}
0.45	(8,4)	3.31×10^9	-3.44×10^9	2.91×10^{14}
0.35	(4,2)	4.28×10^{12}	-7.25×10^{12}	2.84×10^{14}
0.35	(6,2)	4.92×10^{11}	-5.16×10^{11}	2.81×10^{14}
0.35	(8,4)	4.00×10^9	-4.21×10^9	2.79×10^{14}

Table 15: Predictions of M_1, M_2, M_3 in GeV for different (m, n) in Dirac mass matrices, in the normal hierarchical model for case $X = m(1, 1) = 0$. The predicted neutrino mass parameters are $\Delta m_{21}^2 (\Delta m_{23}^2) = 7.5 \times 10^{-5} eV^2 (2.4 \times 10^{-3} eV^2), 7.9 \times 10^{-5} eV^2 (2.59 \times 10^{-3} eV^2), 7.2 \times 10^{-5} eV^2 (2.8 \times 10^{-3} eV^2)$ for $\tan^2 \theta_{12} = 0.50, 0.45, 0.35$ respectively

$\tan^2 \theta_{12}$	(m,n)	\tilde{m}	$(h^\dagger h)_{11}$	κ_1	ϵ_1	Y_B
0.50	(4,2)	4.4×10^{-11}	5.2×10^{-3}	3.6×10^{-3}	5.08×10^{-4}	8.09×10^{-8}
0.50	(6,2)	5.9×10^{-10}	7.0×10^{-3}	1.8×10^{-4}	4.91×10^{-3}	3.93×10^{-8}
0.50	(8,4)	5.9×10^{-10}	6.2×10^{-5}	1.8×10^{-4}	3.88×10^{-5}	3.09×10^{-10}
0.45	(4,2)	4.1×10^{-11}	5.1×10^{-3}	3.5×10^{-3}	4.39×10^{-4}	6.83×10^{-8}
0.45	(6,2)	5.6×10^{-10}	7.5×10^{-3}	1.9×10^{-4}	4.6×10^{-3}	3.90×10^{-8}
0.45	(8,4)	5.6×10^{-10}	6.1×10^{-5}	1.9×10^{-4}	3.68×10^{-5}	3.09×10^{-10}
0.35	(4,2)	3.6×10^{-11}	5.2×10^{-4}	4.2×10^{-3}	2.2×10^{-4}	4.47×10^{-8}
0.35	(6,2)	4.5×10^{-10}	7.3×10^{-3}	2.4×10^{-4}	3.75×10^{-4}	4.03×10^{-8}
0.35	(8,4)	4.5×10^{-10}	6.0×10^{-5}	2.4×10^{-4}	2.98×10^{-5}	3.18×10^{-10}

Table 16: Predictions of ϵ_1 and Y_B for various values of $\tan^2 \theta_{12}$ for normal hierarchical model with Case(ii) $X=0$ presented in Table-15

6 Summary and discussions

We summarise the main points in this work. We give a very brief overview on the phenomenology of the neutrino mass matrices obeying μ - τ symmetry[6]. Different neutrino mass models based on normal as well as inverted hierarchy, are outlined. We then introduce the parametrisation of mass matrices with only two parameters, and their ratio determines the value of the solar mixing angle. The actual values of these parameters are then fixed by the experimental bounds on neutrino mass scales. Such parametrisation not only gives a firm handle on the analysis of the mass matrices but also lowers the solar mixing angle up to the range $\tan^2 \theta_{12} = 0.50 - 0.35$ without affecting atmospheric and Chooz mixing angles from the tribimaximal mixings. The detailed numerical analysis for different forms of mass matrices obeying the μ - τ reflection symmetry, are given in Tables 1-6. However such treatment for degenerate mass matrices are difficult to realise in nature. All the predictions are in excellent agreement with data except inverted hierarchy type B in table 3 where $m_3 \neq 0$. Here Δm_{23}^2 is highly dependent on solar mixing angle, and has best predicted value at around $\tan^2 \theta_{12} \leq 0.45$.

We also address very briefly the stability question of the neutrino mass models under radiative corrections in MSSM, particularly the inverted hierarchy with CP odd in the first two mass eigenvalues. We find that for large $\tan \beta \sim 58 - 60$, the model is not stable under RG running. The evolution of solar mass scale with energy is highly dependent on the input high scale value of solar angle. Solar angle predicted by tribimaximal angle and below, does not lead to stability of the model. Similarly, the evolution of atmospheric mixing angle with energy scale shows sharp decrease for the case $m_3 \neq 0$ condition, making the model unstable. However non-zero value of m_3 maintains the stability of the evolution of solar mass scale. this implies that tribimaximal mixings with inverted hierarchy are not so stable under RG analysis in MSSM. Normal hierarchical models are generally stable under RG analysis in MSSM whereas inverted hierarchy type A models are highly unstable. In a self consistent way we apply these mass matrices for the prediction of baryon asymmetry of the universe via leptogenesis and we find that only normal hierarchical model gives good results consistent with observed data. The three theoretical pieces of predictions presented in the work, show that normal hierarchical model appears to be more favourable in nature than inverted hierarchical models. The parametrisation presented here is by no means unique but the analysis presented here strengthens the foundation of

μ - τ symmetry in neutrino sector, based on realistic GUT models.

Acknowledgements

NNS thanks the High Energy Physics Group, the Abdus Salam ICTP, Trieste, Italy, for kind hospitality during the course of the work.

References

- [1] M. C. Gonzalez-Garcia, Michele Maltoni, Phys.Rept.460(2008)1-129, **ArXiv: 0704.1800**.
- [2] S. T. Petcov, A. Y. Smirnov, Phys. Lett. **B322**, 109 (1994); M. Raidal, Phys. Rev. Lett. **93**, 161801(2004); H. Minakata, A. Y. Smirnov, Phys. Rev. **D70**, 073009(2004); H. Minakata, **hep-ph/0505262**; S. Antusch, S. F. King, R. N. Mohapatra, Phys.Lett. **B618**, 150(2005); C. Jarlskog, Phys. Lett. **B625**, 63(2005); Kathrin A. Hochmuth, Werner Rodejohann, Phys.Rev. **D75**, 073001(2007); B. C. Chauhan, M. Picariello, J. Pulido, E. Torrente-Lujan, Eur. Phys. J. **C50**, 573(2007).
- [3] Michael A. Schmidst, Alexei Yu. Smirnov, Phys. Rev. **D74**, 113003(2006).
- [4] S.F.King arXiv: 0710.0530
- [5] Sanjib Kumar Agarwalla, M. K. Parida, R. N. Mohapatra, G. Rajasekaran, Phys. Rev. **D75**, 033007(2007).
- [6] An incomplete list: P. F. Harrison, W. G. Scott, Phys. Lett. **B547**, 219(2002); C. S. Lam, Phys. Rev. **D71**, 093001(2005); **hep-ph/0503157**; W. Grimus, **hep-ph/0610158**; W. Grimus, L. Lavoura, J. Phys. G34: (2007)1757, **hep-ph/0611149**; A. S. Joshipura, Eur. Phys. J. C53: (2008)77, **hep-ph/0512252**; T. Kitabayashi, M. Yasue, Phys. Lett. **B490**, 236(2000); E. Ma, Phys. Rev. **D70**, 031901(2004); K. S. Babu, R. N. Mohapatra, Phys. Lett. **B532**, 77(2002); T. Fukuyama, H. Nishiura, **hep-ph/9702253**; K. Fuki, M. Yasue, Nucl. Phys. B783, (2007)31, **hep-ph/0608042**; A. Ghoshal, Mod. Phys. Lett. **A19**, 2579(2004); **hep-ph/0304090**; T. Ohlsson,

- G. Seidl, Nucl. Phys. **B643**, 247(2002); Riazuddin, Eur. Phys. J. C51, (2007)699, **arXiv:0707.0912**; Takeshi Fukuyama, arXiv:0804.2107.
- [7] Y. H. Ahn, Sin Kyu Kang, C. S. Kim, Jake Lee, Phys.Rev.D73: (2006)093005, **hep-ph/0602160**.
- [8] Yoshio Koide, Phys. Rev. **D69**, 093001(2004); Yoshio Koide, H. Nishiura, K. Matsuda, T. Kikuchi, T. Fukuyama, Phys. Rev. **D66**, 093006(2002); Koichi Matsuda, H. Nishiura, Phys. Rev. **D73**, 013008(2006).
- [9] Y. Koide, E. Takasugi, Phys. Rev. D77,(2008)016006, **arXiv:0706.4373**.
- [10] R. N. Mohapatra, S. Nasri, Hai-Bo Yu, Phys. Lett. **B636**, 114(2006).
- [11] P. F. Harrison, D. H. Perkins, W.G. Scott, Phys. Lett. **B530**, 167(2002); P. F. Harrison, W. G. Scott, Phys. Lett. **B557**, 76(2003).
- [12] E. Ma, Phys. Rev. **D70**, 03191(2004); Mod. Phys. Lett. **A21**, 2931(2006); S. F. King, Michal Malinsky, Phys. Lett. **B645**, 351(2007); R. N. Mohapatra, s. Nasri, Hai-Bo Yu, Phys. Lett. B639, (2006)318, **hep-ph/0605020**; Xiao-Gang He, A. Zee, Phys. Lett. B645, (2007)427, **hep-ph/0607163**; Florian Plentinger, Werner Rodejohann, Phys. Lett. B625, (2005)264 **hep-ph/0507143**.
- [13] N. Nimai Singh, Monisa Rajkhowa, Abhijit Borah, J. Phys. G: Nucl. Part. Phys. **34**, 345(2007); **hep-ph/0603154**.
- [14] N.Nimai Singh, Monisa Rajkhowa, Abhijit Borah, Pramana J.Physics 69,533(2007).
- [15] K. A. Hochmuth, S. T. Petcov, W. Rodejohann, Phys. Lett. B654, (2007)177, **arXiv:0706.2975**.
- [16] K. S. Babu, Abdel G. Bachri, Zurab Tavartkiladzee, Int. J. Mod. Phys. A23, (2008)1679, **arXiv:0705.4419**.
- [17] Walter Grimus, Anjan S. Joshipura, Satoru Kaneko, L.Lavoura, H. Sawanaka, M. Tanimoto, Nucl. Phys. **B713**, 151(2005).

- [18] Manfred Linder, Alexander Merle, Werner Rodejohann, Phys.Rev. **D73**, 053005(2006).
- [19] Takeshi Fukuyama, arXiv:0804.2107.
- [20] P. H. Frampton, S. L. Glashow, T. Yanagida, Phys. Lett. **B548**, 119(2002).
- [21] Mahadev Patgiri, N. Nimai Singh, Phys. Lett. **B567**, 69(2003).
- [22] Mahadev Patgiri, N. Nimai Singh, Int. J. Mod. Phys. **A18**, 743(2003)
- [23] Javier Ferrandis and Sandip Pakvasa, Phys.Rev. **D71**,033004(2005).
- [24] Aik Hui Chan, Harald Fritzsch, Shu Luo, Zhi-zhong Xing, Phys. Rev. **D76**, (2007)073009, **arXiv:0704.3153**.
- [25] S.Antusch, J. Kersten, M. Linder and M. Ratz, Nucl. Phys.**B674**, 401(2003).
- [26] Amol Dighe, Srubabati Goswami, Probir Roy, Phys.Rev. **D73**, 071301(2006); **arXiv:0704.3735**; Amol Dighe, Srubabati Goswami, Werner Rodejohann, Phys. Rev. **D75**, 073023(2007).
- [27] S. F. King, N. Nimai Singh, Nucl. Phys. **B591**, 3(2003); Nucl.Phys, **B596**, 81(2001).
- [28] M. K. Das, M. Patgiri, N. Nimai Singh, Pramana, **65**, 995(2005); **hep-ph/0407185**.
- [29] P. H. Chankowshi, W. Krolikowski, S. Pokorski, Phys. Lett. **B473**, 109(2000).
- [30] M. K. Parida, N. Nimai Singh, Phys. Rev. **D59**, 032002(1998).
- [31] Sacha Davidson, Enrico Nardi, Yosef Nir, arXiv:0802.2962.
- [32] A. D. Sakharov, JETP Lett.5, 24(1967).
- [33] M. Fukugita and T. Yanagida, Phys. Lett 174B, 45(1986).
- [34] K.S.Babu, A.Bachri, Nucl.Phys.B738(2006)76; Carl H. Albright, S. M. Barr, Phys. Rev. **D70**, 033013(2004).

- [35] T. Hambye, ICTP Summer School Lecture notes (2004).
- [36] A.K. Sarma, H. Zeen Devi, N. Nimai Singh, Nucl. Phys. B765, 142(2007).
- [37] V. A. Kuzmin, v. A. Rubakov, M. E. Shaposhnikov, Pjhs. Lett, 115B, 36(1985).
- [38] Biswajit Adhikary and Ambar Ghosal, arXiv: 0803.3582 and references therein.
- [39] W. Buchmuller, arXiv: 0710.5857.
- [40] E. K. Akhmedov, M. Frigerio and A. Y. Smirnov, JHEP 0309, 021(2003).
- [41] E. W. Kolb, M. S. Turner, The Early Universe, Addison-Wesely, New York (1990).